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Arrhythmia Classification in Long-Term Data Using Relative RR Intervals

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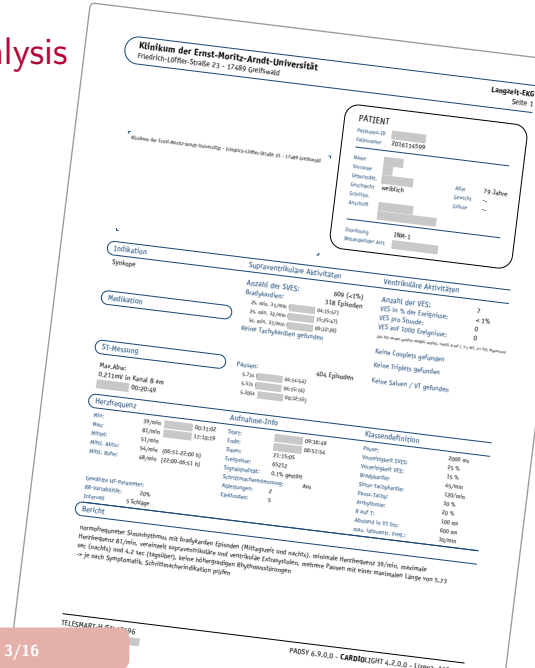
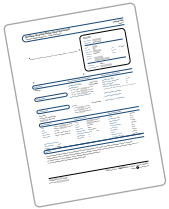
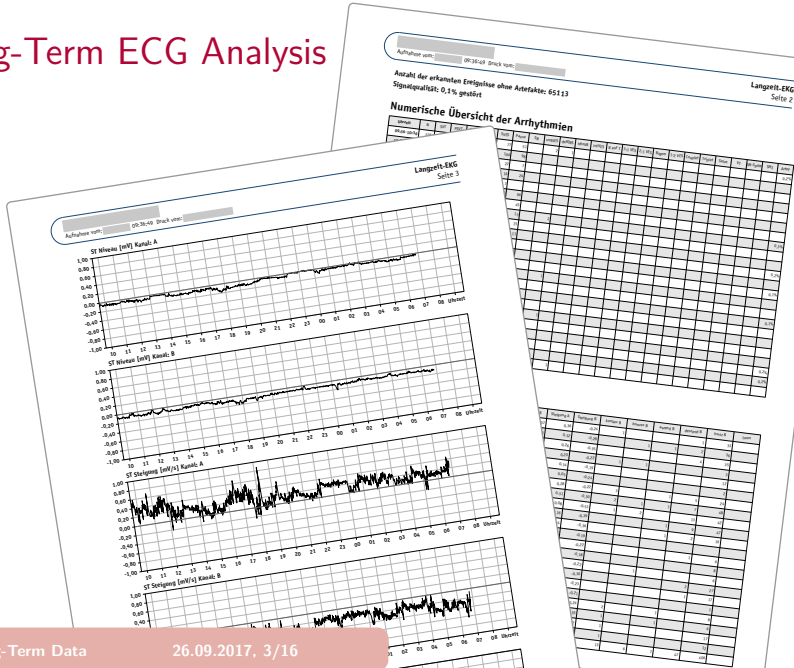
Outline

Standard Output of Long-Term ECG Analysis

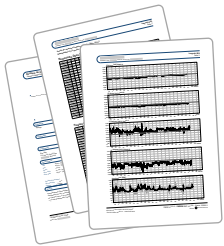


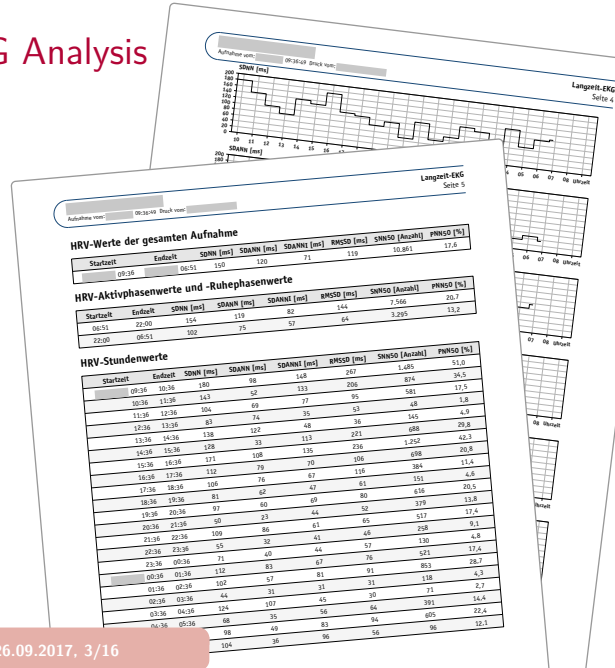
Standard Output of Long-Term ECG Analysis





Standard Output of Long-Term ECG Analysis





Langzeit-EKG
Seite 5

Aufnahme vom: 09:36:49 Druck vom:

HRV-Werte der gesamten Aufnahme

Startzeit	Endzeit	SDNN [ms]	SDANN [ms]	SDANNI [ms]	RMSSD [ms]	SNN50 [Anzahl]	PNN50 [%]
09:36	06:51	150	120	71	119	10.861	17,6

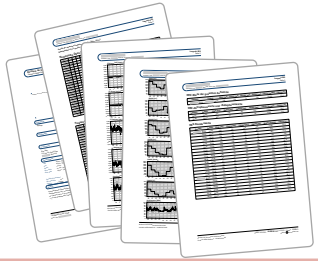
HRV-Aktivphasenwerte und -Ruhephasenwerte

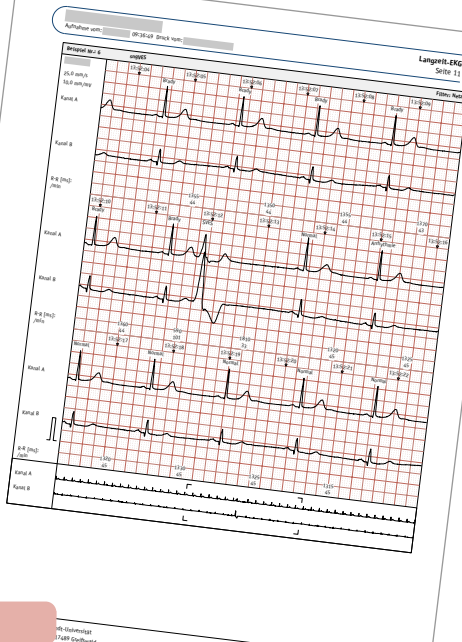
Startzeit	Endzeit	SDNN [ms]	SDANN [ms]	SDANNI [ms]	RMSSD [ms]	SNN50 [Anzahl]	PNN50 [%]
06:51	22:00	154	119	82	144	7.566	20,7
22:00	06:51	102	75	57	64	3.295	13,2

HRV-Stundenwerte

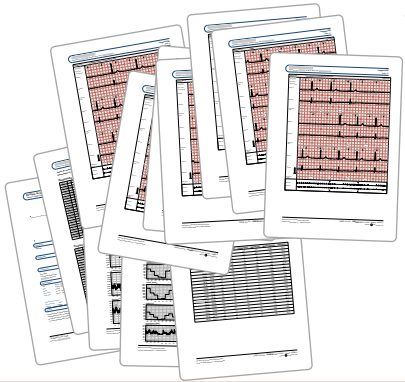
Startzeit	Endzeit	SDNN [ms]	SDANN [ms]	SDANNI [ms]	RMSSD [ms]	SNN50 [Anzahl]	PNN50 [%]
09:36	10:36	180	98	148	267	1.485	51,0
10:36	11:36	143	52	133	206	874	34,5
11:36	12:36	104	69	77	95	581	17,5
12:36	13:36	83	74	53	53	48	1,8
13:36	14:36	138	122	48	36	145	4,9
14:36	15:36	178	33	113	221	688	29,8
15:36	16:36	171	108	135	236	1.252	42,3
16:36	17:36	112	79	70	106	698	20,8
17:36	18:36	106	76	67	116	384	11,4
18:36	19:36	81	62	47	61	151	4,6
19:36	20:36	97	60	69	80	616	20,5
20:36	21:36	50	23	44	52	379	13,8
21:36	22:36	109	86	61	65	517	17,4
22:36	23:36	55	32	41	46	258	9,1
23:36	00:36	71	40	44	57	130	4,8
00:36	01:36	112	83	67	78	521	17,4
01:36	02:36	102	57	81	91	853	28,7
02:36	03:36	44	31	31	31	118	4,3
03:36	04:36	124	107	45	30	71	2,7
04:36	05:36	68	35	56	64	391	14,4
		98	49	83	94	605	22,4
		104	36	96	56	96	12,1

Standard Output of Long-Term ECG Analysis



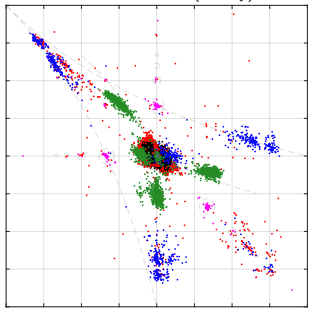


Standard Output of Long-Term ECG Analysis

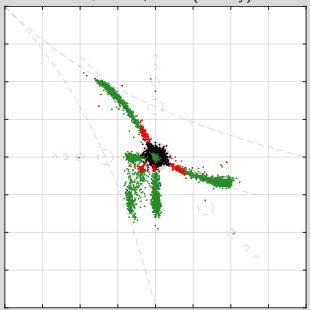


- Is this an efficient way of analyzing long-term data?
- Are spreadsheets and short ECG segments enough for a comprehensive arrhythmia screening?

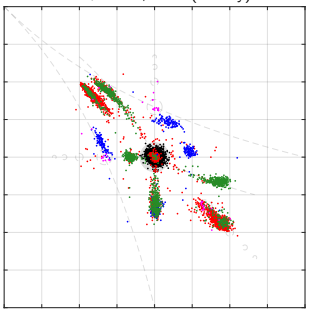
New Visualization Option



N 69068, Q 28805
| 3198, A 969, V 16



N 64645, | 4997
V 2008, Q 50, A 13



N 75725, | 5523
V 2476, Q 156, A 20

N - Normal beat, A - Atrial premature beat, V - Premature ventricular contraction, Q - Unclassifiable beat, | - Isolated QRS-like artifact

Arrhythmias and Relative RR Intervals

Relative RR Intervals

Relative RR intervals rr_i are defined as a weighted difference of RR intervals:

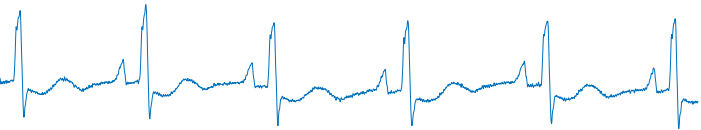
$$rr_i := \frac{RR_i - RR_{i-1}}{\frac{1}{2}(RR_i + RR_{i-1})} \quad \text{for all } i \in \{2, \ldots, n\}$$

Relative RR intervals have the following properties:

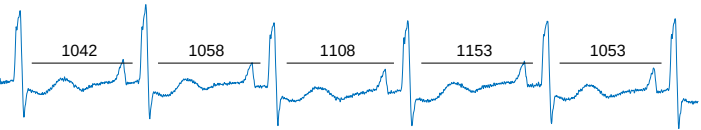
1. $-2 \leq rr_i \leq +2$
2. $rr_i = 0$ if and only if $RR_i = RR_{i-1}$
3. $rr_i = -2$ if and only if $RR_i = 0$
4. $rr_i = +2$ if and only if $RR_{i-1} = 0$

Interpolated extrasystole - isolated

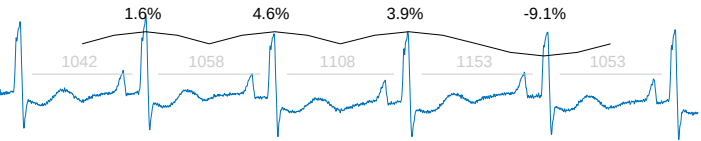
Interpolated extrasystole - isolated



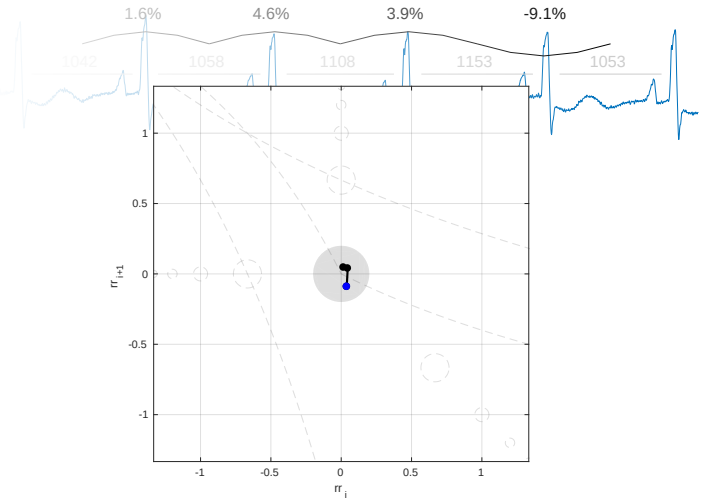
Interpolated extrasystole - isolated



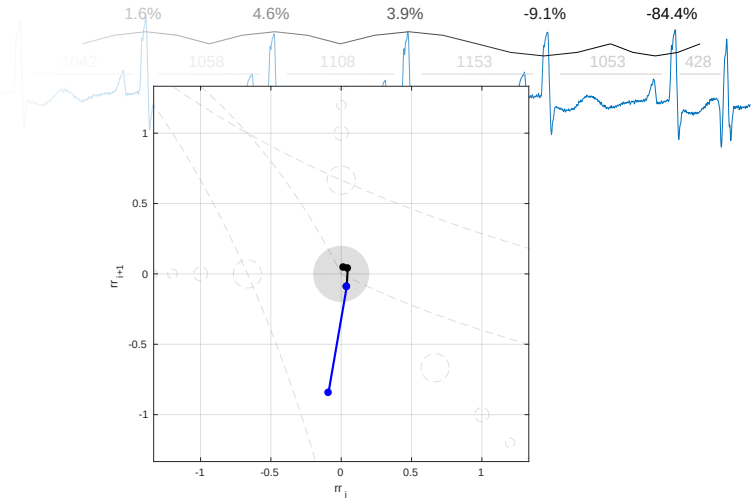
Interpolated extrasystole - isolated



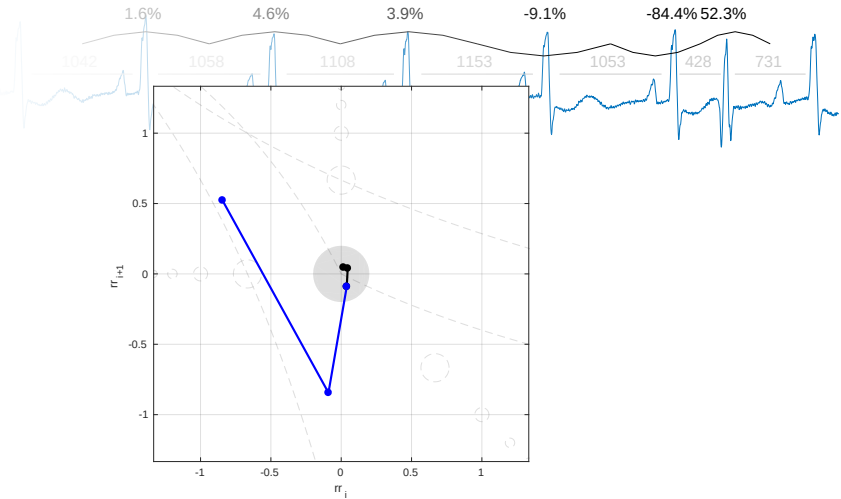
Interpolated extrasystole - isolated



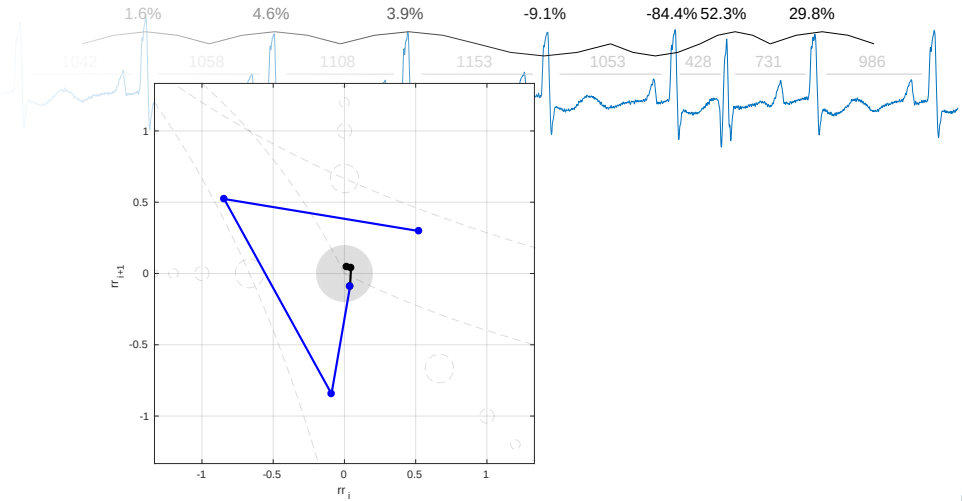
Interpolated extrasystole - isolated



Interpolated extrasystole - isolated



Interpolated extrasystole - isolated



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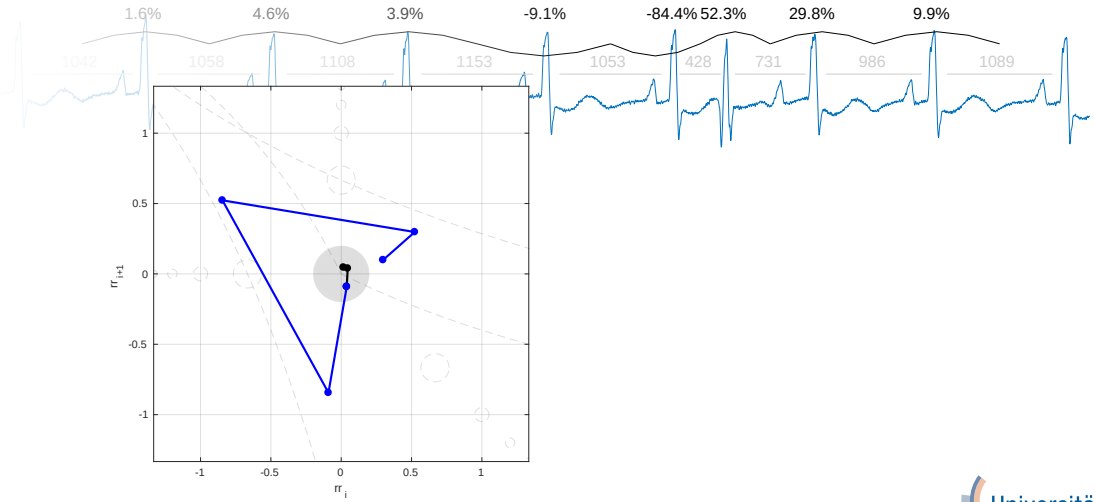
Interpolated extrasystole - isolated



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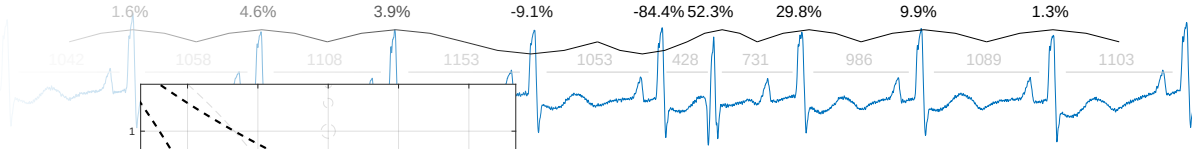
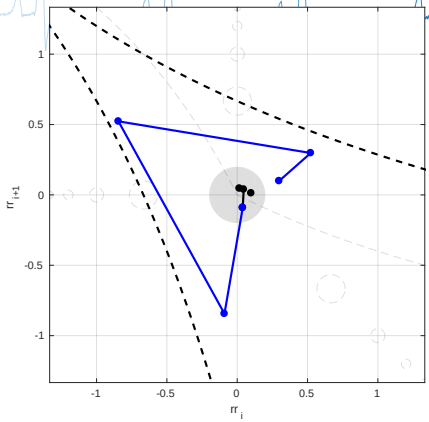
Interpolated extrasystole - isolated





$$f_1 : rr_{i+1} = 6 - \frac{16}{2 - rr_i}$$

$$f_2 : rr_{i+1} = 1 - \frac{2 + 3 \cdot rr_i}{6 + rr_i}$$

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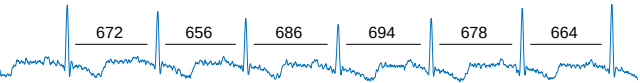
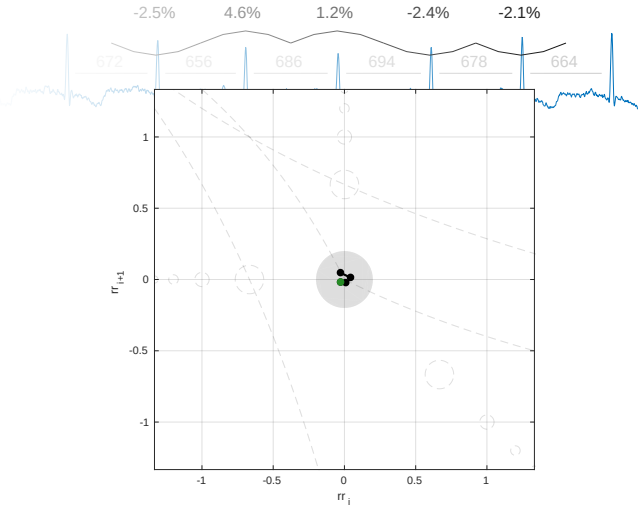
Extrasytole with compensatory pause - isolated PVC

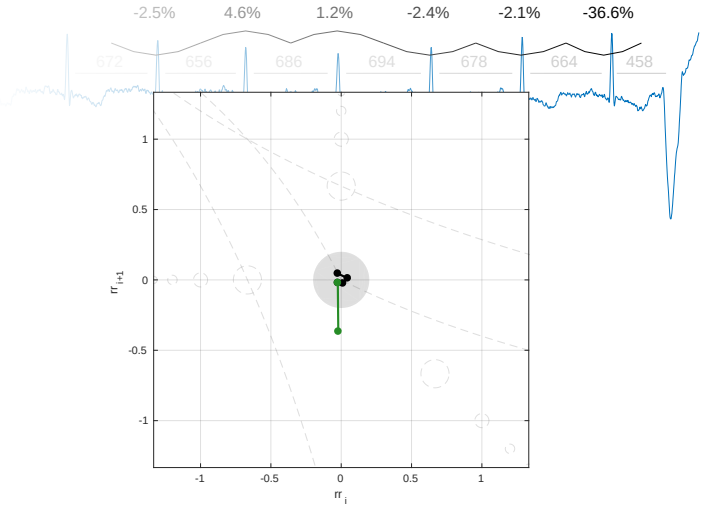
Extrasytole with compensatory pause - isolated PVC



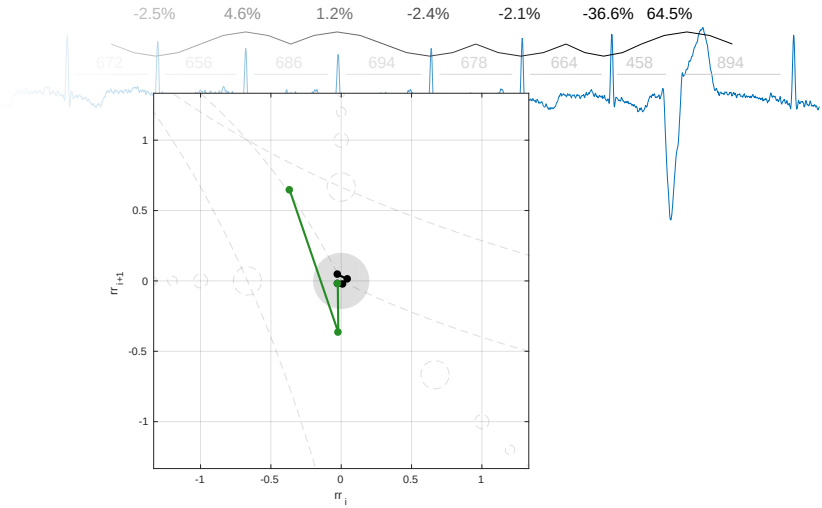
Extrasytole with compensatory pause - isolated PVC



Extrasytole with compensatory pause - isolated PVC



Extrasytole with compensatory pause - isolated PVC



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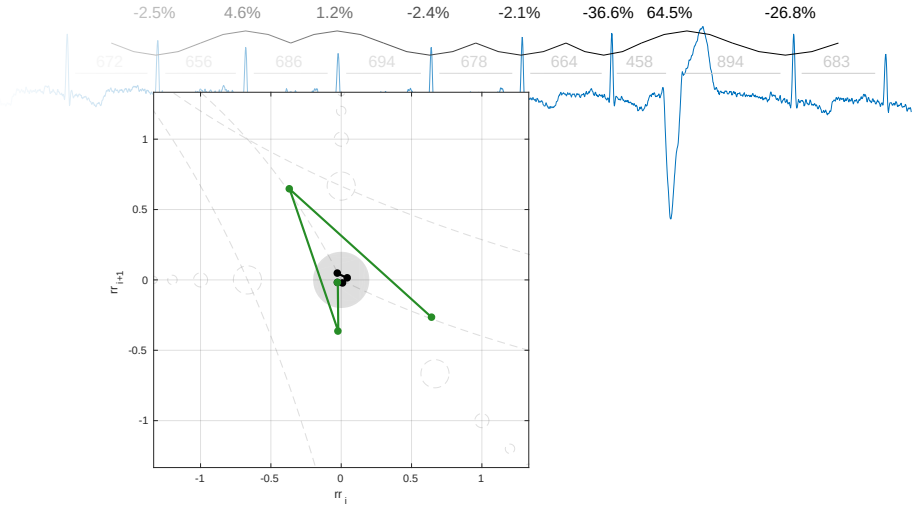
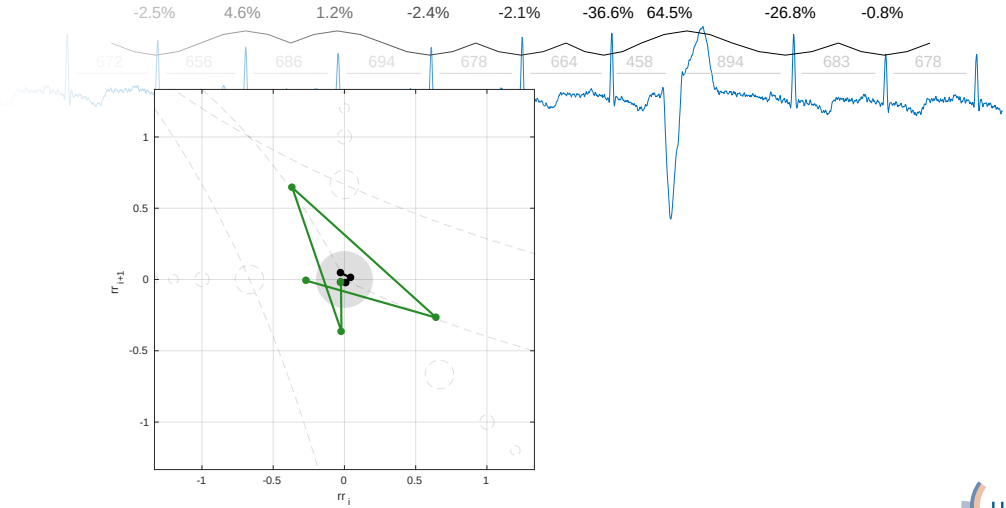
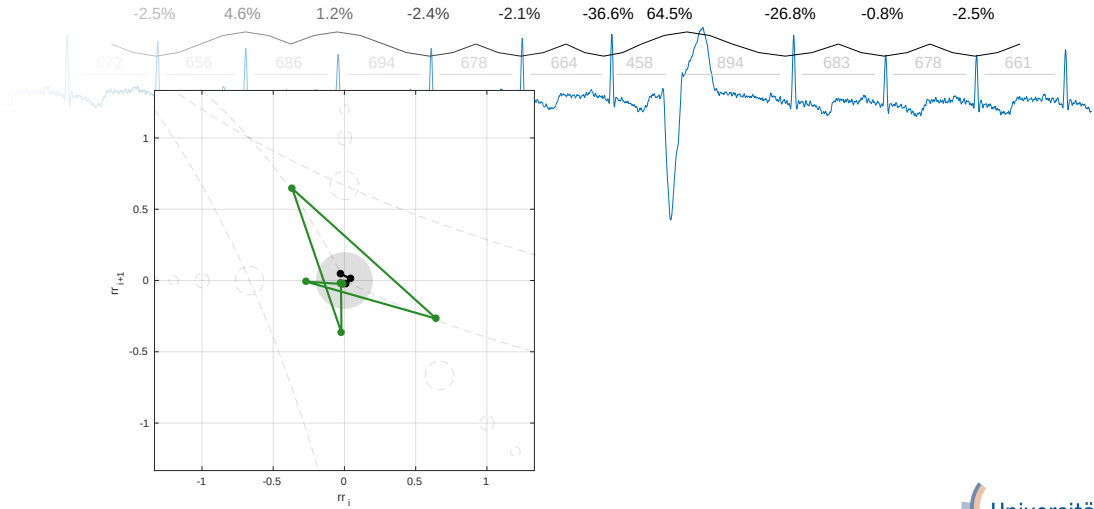
Extrasytole with compensatory pause - isolated PVC



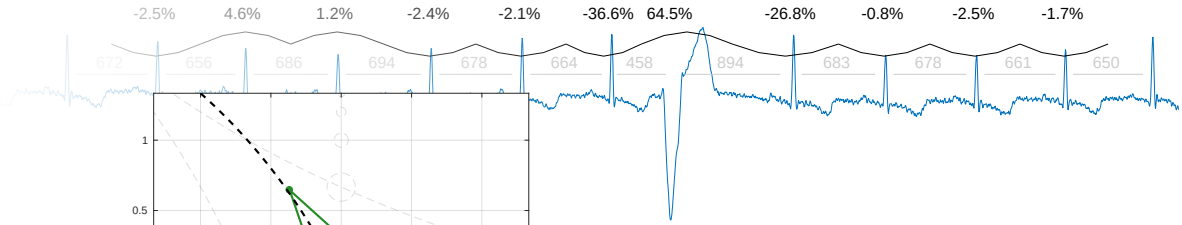
Extrasytole with compensatory pause - isolated PVC

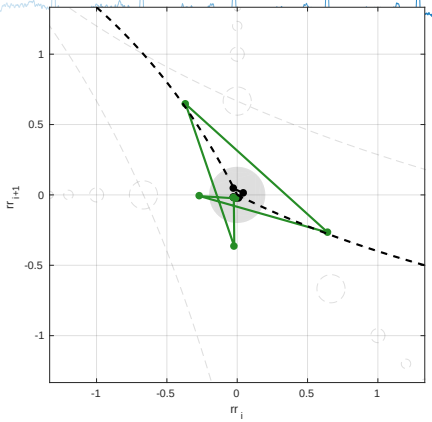


Extrasytole with compensatory pause - isolated PVC



Extrasytole with compensatory pause - isolated PVC



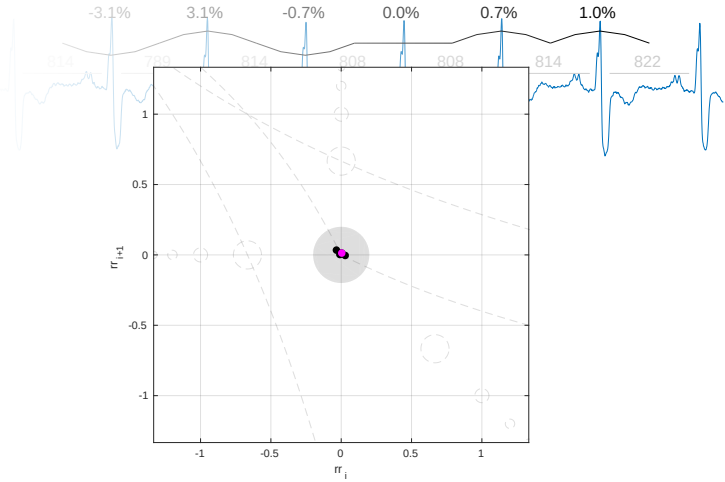


$$f_3 : rr_{i+1} = +4 - \frac{8}{2 - rr_i}$$

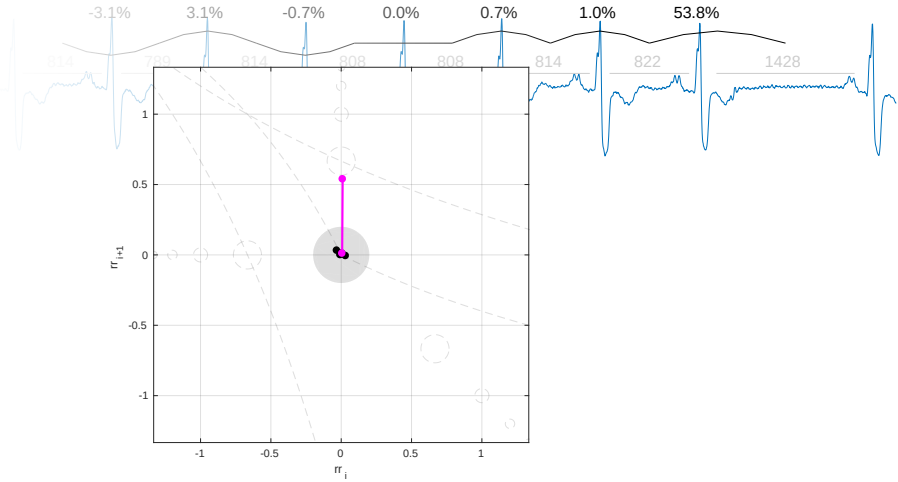
$$f_4 : rr_{i+1} = -2 + \frac{8}{4 + rr_i}$$

Skipped/blocked beats

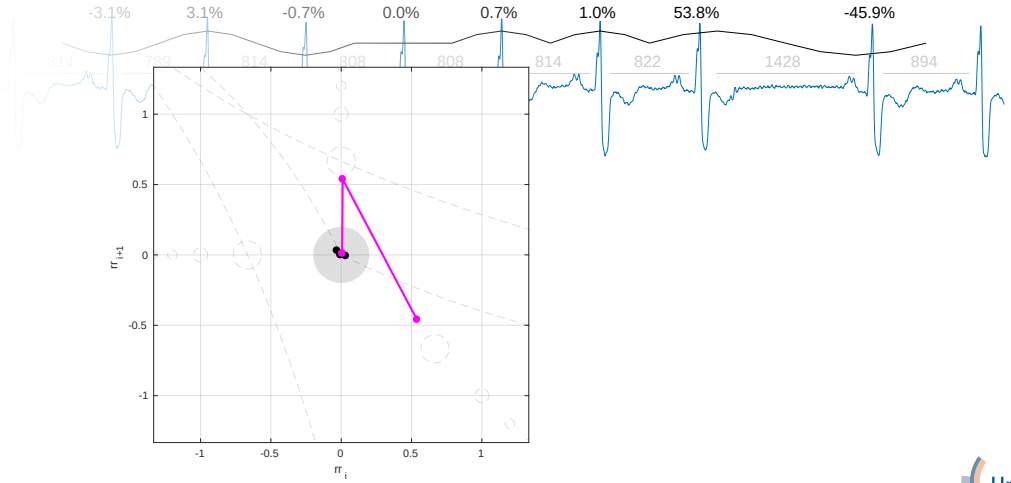
Skipped/blocked beats



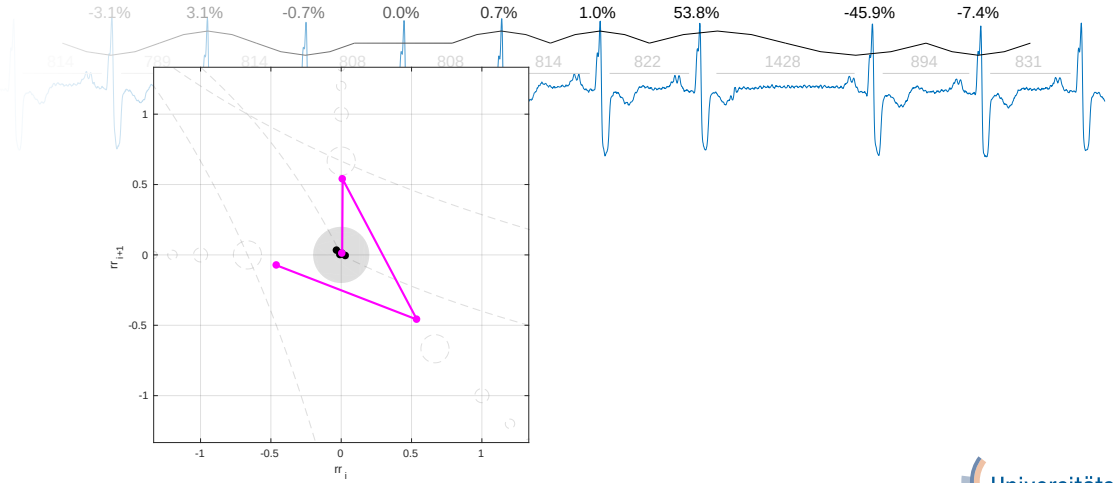
Skipped/blocked beats



Skipped/blocked beats



Skipped/blocked beats



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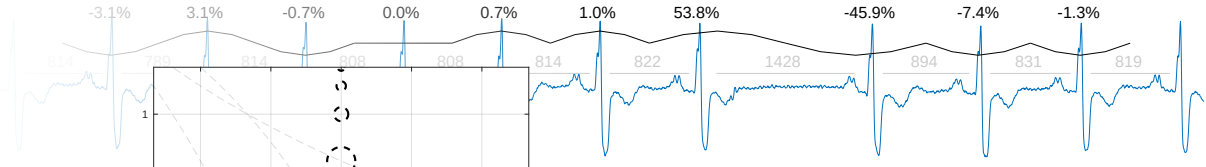
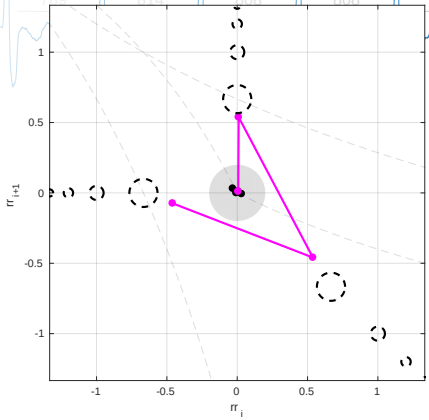
Skipped/blocked beats



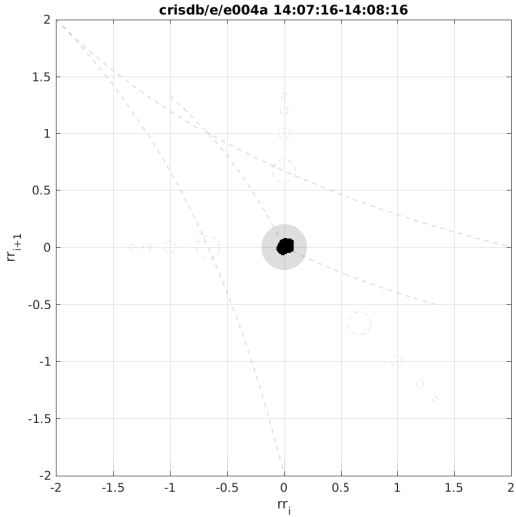


$f_5 : rr_i = \frac{2}{1+2/j}$, j skipped beats

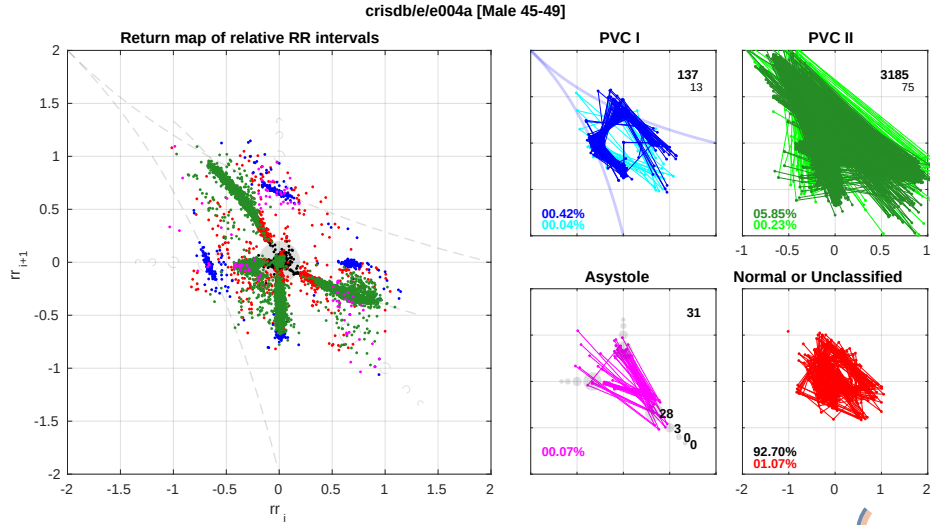
$f_6 : rr_{i+1} = -rr_i$

Visual Inspection of Long-Term Records

Long-term ECG analysis - time lapse



Long-term ECG analysis - complete view



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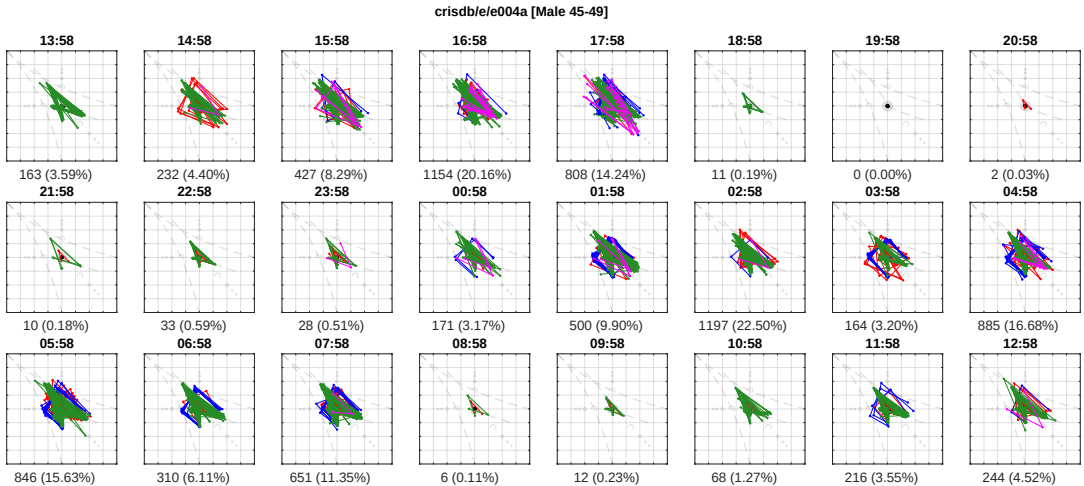
Long-term ECG analysis - hourly view



Outlook

Recognition, diagnosis and management of arrhythmias

- Characterize and classify patients
- Opportunity for diagnostics
- Screening and case finding
- Discriminate artifacts from arrhythmias

Thank You for Your Attention!

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Appendix

CAST Database

Cardiac Arrhythmia Suppression Trial (CAST)

CAST RR Interval Sub-Study Database available on Physionet.org

Hypothesis: Suppression of asymptomatic or mildly symptomatic ventricular premature complexes (PVCs) in survivors of myocardial infarction (MI) would decrease the number of deaths from ventricular arrhythmias and improve survival.
Enrollment: Acute MI within the preceding 2 years and 6 or more PVCs per hour during a pre-treatment long-term ECG recording.
Intervention: Patients randomly assigned to receive antiarrhythmic drugs (encainide 1987-89, flecainide 1987-89, moricizine 1987-91) or a placebo. (Double-Blind)
Results: Drugs successfully reduced the amount of PVCs but led to more arrhythmia-related deaths (sudden cardiac death).
Free data: For 734 subjects, separate baseline (pre-treatment) and on-therapy records available. Database consists of 1543 records (24h-Holter, $\sim$150 million RR intervals).

Pattern of some Arrhythmia Types

N-NVN-N-N-N-N	isolated, interpolated	▸ mitdb/108
N-NVNVNVNVNVN	bigeminy, interpolated	
N-NVN-NVN-NVN	trigeminy, interpolated	
N-NV--N-N-N-N	isolated, compensatory pause	▸ mitdb/100
N-NV--NV--NV-	bigeminy, compensatory pause	▸ mitdb/233
N-NV--N-NV--N	trigeminy, compensatory pause	▸ mitdb/210
N-NV-N-N-N-N-	isolated, incomplete comp. pause	
N-NVV-N-N-N-N	doublet	▸ mitdb/215
N-NVVVV-N-N-N	multifocal PVC, ventricular tachycardia	▸ mitdb/205
N-N---N-N-N-N	skipped beats / asystole	▸ mitdb/219 AFIB
N-Nx--N-N-N-N	blocked PAC	▸ mitdb/118

Pattern of some Arrhythmia Types

N-NVN-N-N-N-N	isolated, interpolated	▸ mitdb/108	PVC I
N-NVNVNVNVNVN	bigeminy, interpolated		
N-NVN-NVN-NVN	trigeminy, interpolated		
N-NV--N-N-N-N	isolated, compensatory pause	▸ mitdb/100	PVC II
N-NV--NV--NV-	bigeminy, compensatory pause	▸ mitdb/233	
N-NV--N-NV--N	trigeminy, compensatory pause	▸ mitdb/210	
N-NV-N-N-N-N-	isolated, incomplete comp. pause		
N-NVV-N-N-N-N	doublet	▸ mitdb/215	
N-NVVVV-N-N-N	multifocal PVC, ventricular tachycardia	▸ mitdb/205	
N-N---N-N-N-N	skipped beats / asystole	▸ mitdb/219 AFIB	Asystole
N-Nx--N-N-N-N	blocked PAC	▸ mitdb/118	

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVN-N-N` isolated, interpolated

RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 1-a & 1 & 1\end{pmatrix}$ where n is the normal RR interval length

$$
\begin{aligned}
rr_2 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0 \\
rr_3 &= 2 \cdot \tfrac{a-1}{a+1} &&= 2 \cdot \frac{a+1-2}{a+1} = 2 - \frac{4}{a+1} \\
rr_4 &= 2 \cdot \tfrac{(1-a)-a}{(1-a)+a} &&= 2 \cdot (1-2a) = 2 - 4a \\
rr_5 &= 2 \cdot \tfrac{1-(1-a)}{1+(1-a)} &&= 2 \cdot \frac{a}{2-a} = \frac{2}{\frac{2}{a}-1} \\
rr_6 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVN-N-N` isolated, interpolated

Relative RR interval sequence: $\begin{pmatrix} 0 & 2-\frac{4}{a+1} & 2-4a & \frac{2}{\frac{2}{a}-1} & 0 \end{pmatrix}$

Some relations of successive relative RR intervals:

$$
\begin{aligned}
rr_3 = \quad & 2-\tfrac{4}{a+1} && \Leftrightarrow a = \frac{4}{2-rr_3} - 1 \\
rr_4 = \quad & 2-4a && = 6 - \frac{16}{2-rr_3} = 2 \cdot \frac{2+3rr_3}{-2+rr_3} \\
rr_5 = \quad & \tfrac{2}{\frac{2}{a}-1} && = \frac{2}{\frac{8}{2-rr_4}-1} = 2 \cdot \frac{2-rr_4}{6+rr_4}
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVN-N-N` isolated, interpolated

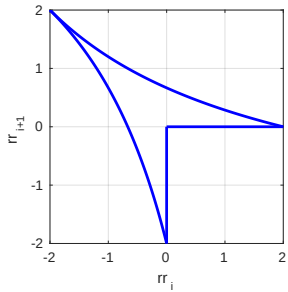
Relative RR interval sequence: $\left(0 \quad 2-\frac{4}{a+1} \quad 2-4a \quad \frac{2}{\frac{2}{a}-1} \quad 0\right)$

Some relations of successive relative RR intervals:

$$rr_3 = 2-\frac{4}{a+1} \Leftrightarrow a = \frac{4}{2-rr_3} - 1$$

$$rr_4 = 2-4a = 6 - \frac{16}{2-rr_3} = 2 \cdot \frac{2+3rr_3}{-2+rr_3}$$

$$rr_5 = \frac{2}{\frac{2}{a}-1} = \frac{2}{\frac{8}{2-rr_4}-1} = 2 \cdot \frac{2-rr_4}{6+rr_4}$$



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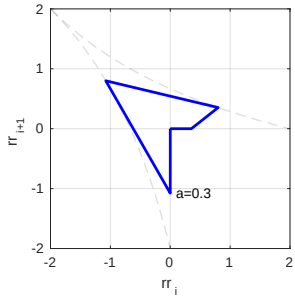

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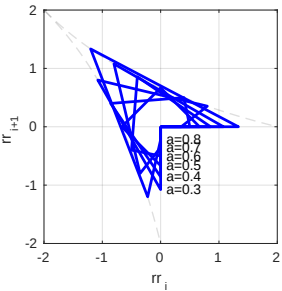

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVNVNVN-N-N` bigeminy, interpolated
RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 1-a & a & 1-a & a & 1-a & 1 & 1\end{pmatrix}$
rr sequence: $\begin{pmatrix}0 & A & B & -B & B & -B & B & C & 0\end{pmatrix}$

$$
\begin{aligned}
A &= 2 - \frac{4}{a+1} \\
B &= 2 - 4a \\
C &= \frac{2}{\frac{2}{a} - 1}
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVNVNVN-N-N` bigeminy, interpolated

RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 1-a & a & 1-a & a & 1-a & 1 & 1\end{pmatrix}$

rr sequence: $\begin{pmatrix}0 & A & B & -B & B & -B & B & C & 0\end{pmatrix}$

$$A = 2 - \frac{4}{a+1}$$
$$B = 2 - 4a$$
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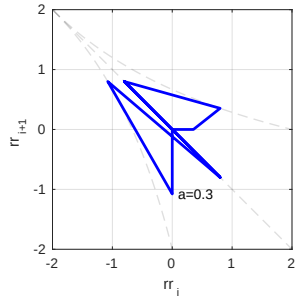


Relative RR Intervals and ectopic beats (PVCI)

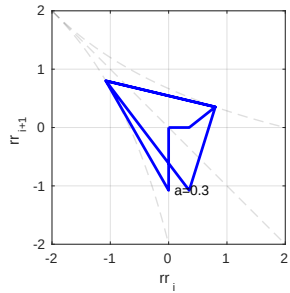
Arrhythmia type: `N-N-NVN-NVN-N` trigeminy, interpolated
RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 1-a & 1 & a & 1-a & 1 & 1\end{pmatrix}$
rr sequence: $\begin{pmatrix}0 & A & B & C & A & B & C & 0\end{pmatrix}$

$$
\begin{aligned}
A &= 2 - \frac{4}{a+1} \\
B &= 2 - 4a \\
C &= \frac{2}{\frac{2}{a} - 1}
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCI)

Arrhythmia type: `N-N-NVN-NVN-N` trigeminy, interpolated
RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 1-a & 1 & a & 1-a & 1 & 1\end{pmatrix}$
rr sequence: $\begin{pmatrix}0 & A & B & C & A & B & C & 0\end{pmatrix}$

$$
\begin{aligned}
A &= 2 - \frac{4}{a+1} \\
B &= 2 - 4a \\
C &= \frac{2}{\frac{2}{a} - 1}
\end{aligned}
$$



Relative RR Intervals and ectopic beats (PVCII)

Arrhythmia type: `N-N-NV--N-N-N` isolated, compensatory pause
RR sequence: $n^*\begin{pmatrix}1 & 1 & a & 2-a & 1 & 1\end{pmatrix}$ where n is the normal RR interval length

$$
\begin{aligned}
rr_2 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0 \\
rr_3 &= 2 \cdot \tfrac{a-1}{a+1} &&= 2 - \frac{4}{a+1} \\
rr_4 &= 2 \cdot \tfrac{(2-a)-a}{(2-a)+a} &&= 2 - 2a \\
rr_5 &= 2 \cdot \tfrac{1-(2-a)}{1+(2-a)} &&= 2 \cdot \frac{a-1}{3-a} = -2 + \frac{4}{3-a} \\
rr_6 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCII)

Arrhythmia type: `N-N-NV--N-N-N` isolated, compensatory pause
Relative RR interval sequence: $\begin{pmatrix} 0 & 2-\frac{4}{a+1} & 2-2a & -2+\frac{2}{3-a} & 0 \end{pmatrix}$

Some relations of successive relative RR intervals:

$$
\begin{aligned}
rr_3 &= 2-\tfrac{4}{a+1} && \Leftrightarrow a = \frac{4}{2-rr_3} - 1 \\
rr_4 &= 2-2a && = 4 - \frac{8}{2-rr_3} = \frac{rr_3}{-2+rr_3} \\
rr_5 &= -2+\tfrac{4}{3-a} && = -2 + \frac{8}{4+rr_4}
\end{aligned}
$$

Relative RR Intervals and ectopic beats (PVCII)

Arrhythmia type: `N-N-NV--N-N-N` isolated, compensatory pause
Relative RR interval sequence: $\begin{pmatrix} 0 & 2-\frac{4}{a+1} & 2-2a & -2+\frac{2}{3-a} & 0 \end{pmatrix}$

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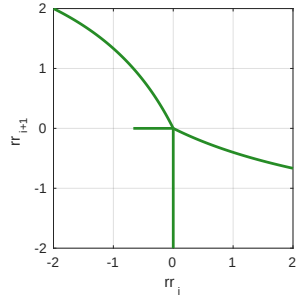
rr_5 &= -2+\tfrac{4}{3-a} \quad = -2 + \frac{8}{4+rr_4}
\end{aligned}
$$



Relative RR Intervals and ectopic beats (PVCII)

Arrhythmia type: `N-N-NV--N-N-N` isolated, compensatory pause
Relative RR interval sequence: $\begin{pmatrix} 0 & 2-\frac{4}{a+1} & 2-2a & -2+\frac{2}{3-a} & 0 \end{pmatrix}$

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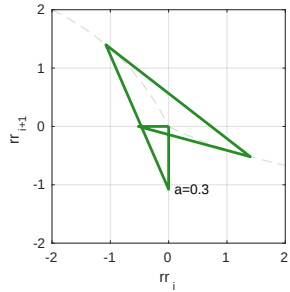

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Arrhythmia type: `N-N-NV--N-N-N` isolated, compensatory pause
Relative RR interval sequence: $\begin{pmatrix} 0 & 2-\frac{4}{a+1} & 2-2a & -2+\frac{2}{3-a} & 0 \end{pmatrix}$

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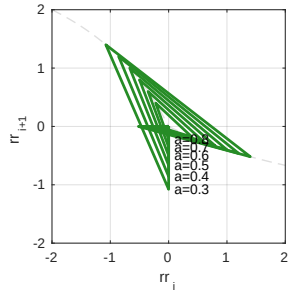
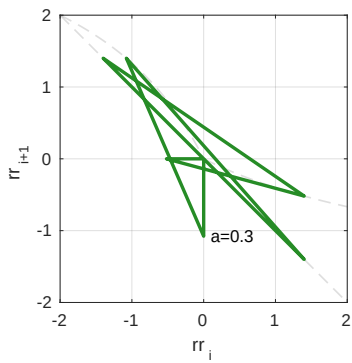
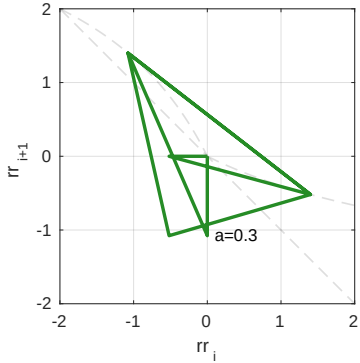


Relative RR Intervals and ectopic beats (PVCII)

Arrhythmia type: `N-N-NV--NV--NV--N` bigeminy, compensatory pause
Arrhythmia type: `N-N-NV--N-NV--N-N` trigeminy, compensatory pause





Relative RR Intervals and ectopic beats (Asystole)

Arrhythmia type: `N-N-N---N-N-N` skipped beats
RR sequence: $n^*(1 \quad 1 \quad 1+k \quad 1 \quad 1)$ where k is the number of skipped beats

$$
\begin{aligned}
rr_2 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0 \\
rr_3 &= 2 \cdot \tfrac{(1+k)-1}{(1+k)+1} &&= 2 \cdot \frac{k}{2+k} \\
rr_4 &= 2 \cdot \tfrac{1-(1+k)}{1+(1+k)} &&= 2 \cdot \frac{-k}{2+k} \\
rr_5 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0
\end{aligned}
$$

Relative RR Intervals and ectopic beats (Asystole)

Arrhythmia type: `N-N-N---N-N-N` skipped beats
RR sequence: $n^*(1 \quad 1 \quad 1+k \quad 1 \quad 1)$ where k is the number of skipped beats

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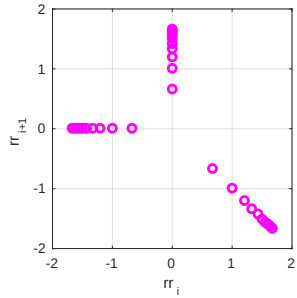
rr_4 &= 2 \cdot \tfrac{1-(1+k)}{1+(1+k)} &&= 2 \cdot \frac{-k}{2+k} \\
rr_5 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0
\end{aligned}
$$



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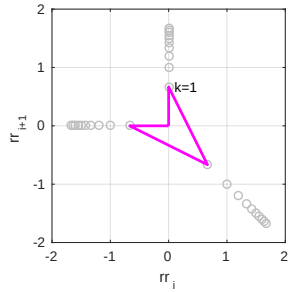
rr_4 &= 2 \cdot \tfrac{1-(1+k)}{1+(1+k)} &&= 2 \cdot \frac{-k}{2+k} \\
rr_5 &= 2 \cdot \tfrac{1-1}{1+1} &&= 0
\end{aligned}
$$



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\end{aligned}
$$